Small Sample Evidence on the Impact of Generated Variables in Event Studies

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Abstract: This paper provides some small sample evidence on the appropriateness of ordinary least squares (OLS) and instrumental variable (IV) estimators of the structural equation, and on the appropriate method of performing hypothesis testing in event studies when generated variables are present. In event studies, the number of observations used to estimate the auxiliary equations (and compute the generated variables) and the structural equation can differ quite substantially. In certain circumstances, this means the appropriate estimator of the structural equation is the IV estimator rather than OLS estimator. Some Monte Carlo suggests that an IV estimator of the parameters of interest can lead to considerably smaller biases than the biases of the OLS estimator. Sizes and powers of tests associated with the coefficient of the generated variable do not seem to be affected by the presence of the generated variable. In contrast, the sizes of tests associated with the constant are considerably distorted when the generated variable should be included in the structural equation.

Keywords: Bias; Event studies; Generated variables; Hypothesis testing; Power; Size

1. INTRODUCTION

In the typical examples of generated variables in economics, the presence of generated variables leads to errors that are serially correlated and heteroscedastic causing estimators ignoring the generated variables problem to be inefficient and have problems with hypothesis testing [Pagan, 1986]. To rectify the problems with hypothesis testing, Smith and McAleer's [1994] Monte Carlo evidence indicates that it is preferable to use test statistics computed using the known form of the covariance matrix of the estimators rather than using Newey-West's estimate of the covariance matrix. In event studies, the presence of generated variables usually only causes heteroscedasticity. However, the number of observations used to estimate the auxiliary equations (and compute the generated variables) and the structural equation can differ quite substantially. In certain circumstances, this means the appropriate estimator of the structural equation is the instrumental variable (IV) estimator rather than the ordinary least squares (OLS) estimator [see McKenzie and McAleer, 1998]. The purpose of this paper is to provide some small sample evidence in an event study context on: (a) the appropriateness of OLS and IV estimators of the structural equation; and (b) the appropriate method to take account of heteroscedasticity when performing hypothesis testing. It is found that the biases of OLS are quite considerable compared to an IV estimator. The results for hypothesis testing suggest that the impact of generated variables differs depending on the parameter involved in the hypothesis test.

2. EVENT STUDIES

The equations estimated in a typical event study are based on the market model. In the estimation window, this model can be stylised as follows:

$$y_{it} = \alpha_i + \beta_i x_{it} + e_{it}, i = 1,.., N, t = 1,.., T, \quad (1)$$

where y_{it} is the rate of return on the ith firm's equity at time t, x_{it} is the rate of return on the market portfolio at time t for the ith firm, and e_{it} is an error term which is assumed to be independently distributed with zero mean and variance σ_{ei}^2. In the event window, it is assumed that

$$y_{iT+1} = \alpha_i + \beta_i x_{iT+1} + \delta_1 + \delta_2 Z_i + e_{iT+1}, \quad i = 1,.., N, \quad (2)$$

where Z_i is a characteristic of firm i. In (2), the null hypotheses of interest are $\delta_1 = 0$ and $\delta_2 = 0$, that is, the announcement at time T+1 has no impact on the firm's rate of return.

Typically, (1) is estimated by OLS for each i to obtain estimates of α_i and β_i, A_i and B_i. These estimates are then used to rewrite (2) as

$$y_{iT+1} - A_i - B_i x_{iT+1} = \delta_1 + \delta_2 Z_i + e_{iT+1} + (\alpha_i - A_i) + (\beta_i - B_i) x_{iT+1}, i = 1,.., N. \quad (3)$$

Equation (3) (or (2)) is referred to as the structural equation and (1) is referred to as the auxiliary equation (or first stage model). When Z_i is observed, this is a standard event study model. However, as McKenzie and McAleer [1998] observe, quite often the explanatory variables used in event studies are generated in some way. In this paper, analysis is focused on the case where $Z_i = \beta_i$ [see McKenzie and McAleer, 1998, Table 1 for some examples]. Since the explanatory variable Z_i is unobservable, it needs to be estimated say as B_i. In this case, (3) can be rewritten as

$$y_{iT+1} - A_i - B_i x_{iT+1} = \delta_1 + \delta_2 B_i + \lambda_{iT+1}, \quad (4)$$

$$\lambda_{iT+1} = e_{iT+1} + (\alpha_i - A_i) + (\beta_i - B_i)(x_{iT+1} + \delta_2). \quad (5)$$

It should be noted that the regressor in (4) will be correlated with the error term given in (5) since

$$E(B_i \lambda_{iT+1}) = \sigma_{ei}^2 (x_{iT+1} + \delta_2 - \bar{x}_i) / V_{xi}, \quad (6)$$

where $\bar{x}_i = \sum_{t=1}^{T} x_{it} / T$, and $V_{xi} = \sum_{t=1}^{T} (x_{it} - \bar{x}_i)^2$.

As $T \to \infty$, $V_{xi} \to \infty$ so that $E(B_i \lambda_{iT+1}) \to 0$. That is, the correlation disappears as the number of observations used at the first stage goes to infinity.

In addition, λ_{iT+1} is heteroscedastic with variance

$$E(\lambda_{iT+1}^2) = \sigma_{ei}^2 [1 + (1/T) + (x_{iT+1} + \delta_2 - \bar{x}_i)^2 / V_{xi}]. (7)$$

As $T \to \infty$, $E(\lambda_{iT+1}^2) \to \sigma_{ei}^2$. That is, the form of the heteroscedasticity simplifies greatly as the number of observations used at the first stage goes to infinity. As δ_2 increases in size, both the correlation between the regressors and the error in (4), and the degree of the heteroscedasticity can be expected to increase.

Given the heteroscedasticity of the error in (4), it is natural to consider a GLS transformation of (4):

$$(y_{iT+1} - A_i - B_i x_{iT+1}) / w_i = \delta_1 / w_i + \delta_2 B_i / w_i + \lambda_{iT+1} / w_i. \quad (8)$$

Three choices of w_i are considered: (A) σ_{ei}; (B) $\sigma_{ei} [1 + (1/T) + (x_{iT+1} - \bar{x}_i)^2 / V_{xi}]^{1/2}$; and (C) $\sigma_{ei} [1 + (1/T) + (x_{iT+1} + \delta_2 - \bar{x}_i)^2 / V_{xi}]^{1/2}$. Choice (A) ignores both the heteroscedasticity arising from the presence of generated variables in both the dependent and explanatory variables. Choice (B) ignores the heteroscedasticity arising from the presence of generated variables in the explanatory variables. OLS applied to (8) with one of these three w_i is referred to as GLS1, GLS2 and GLS3, respectively. The required estimate of σ_{ei} is obtained from the OLS estimates of (1), and the OLS estimate of δ_2 from (4) is used to compute GLS3.

The correlation between the regressors and the error term in (4) prompted McKenzie and McAleer [1998] to suggest that it may be more appropriate to estimate (4) using an IV estimator rather than OLS. The difficulty with IV estimation is finding an appropriate instrument for B_i. In their empirical example, McKenzie and McAleer [1998] use the rank of B_i as an instrument. This estimator is denoted as IV1. Here, an estimate of β_i based on a sample prior to the estimation window is also used, and this estimator is denoted by IV2. IV is also applied to the three choices of w_i in (8) for the two sets of instruments to give estimators denoted as IV1-G1, IV1-G2, IV1-G3, IV2-G1, IV2-G2, and IV2-G3, respectively.

Variances of the OLS estimator are also computed assuming homoscedasticity, and heteroscedasticity with the assumed variance of the errors in (4) being:

(A) σ_{ei}^2; (B) $\sigma_{ei}^2[1+(1/T)+(x_{iT+1}-\bar{x}_i)^2/V_{xi}]$; and

(C) $\sigma_{ei}^2[1+(1/T)+(x_{iT+1}+\delta_2-\bar{x}_i)^2/V_{xi}]$. Tests using these variance estimators are denoted HOM, HET1, HET2 and HET3, respectively. Heteroscedastic-consistent estimates of the variances of the OLS estimator are also computed using White's estimator and tests using this estimator are denoted WHITE. For IV1 and IV2, corresponding estimates of the variances are also used computed to compute test statistics.

3. MONTE CARLO EXPERIMENT

In examining the finite sample performance of estimators and test statistics used in event studies, it is quite common to use actual returns data for both the firm's return and the market return [see Binder, 1998]. In contrast, in this paper the data are generated artificially.

The market returns, x_{it}, are generated as a first-order autoregression

$$x_{it} = \rho x_{it-1} + v_{it},$$

with $v_{it} \sim \text{niid}(0, \sigma_v^2)$. The values of ρ and σ_v^2 are set at $\rho = 0.2$ and $\sigma_v^2 = 1.0$ to loosely replicate the daily returns on the Japanese Nikkei index in 1996. In (1), $\alpha_i = 0\ \forall i$, β_i are generated from a uniform distribution over the range (0,1), and σ_{ei}^2 are generated from a uniform distribution over the range (0.5, 1.0). In any one experiment, the values of β_i, σ_{ei}^2 and x_{it} are fixed. Observations on y_{it} are generated for i=1,..,N according to (1) for t=-(T-1),..,T, and according to (2) with $Z_i = \beta_i$ for t=T+1 assuming the e_{it} are normally distributed. Observations t=-(T-1),..,0 are used to obtain the estimates of β_i used as instruments in the estimator IV2. The observations t=1,..,T are used as the estimation window. In (2), $\delta_1 = 0.0$ and δ_2 takes the values 0.0, 0.1, 0.5, 1.0 and 5.0. The number of observations was varied as N=30, 60, 100, and T=30, 60. For each experiment, the number of replications is 5000. Therefore, the maximum standard errors of the type 1 errors and rejection frequencies are $[0.5(1-0.5)/5000]^{0.5}$=0.007. The nominal sizes of all tests are set equal to 5%.

4. RESULTS

Table 1 presents estimates of the biases of various estimators of δ_2 for various values of δ_2, N and T. Results for GLS2, IV1-G2 and IV2-G2 are not presented because they are very similar to the results for GLS1, IV1-G1 and IV2-G1, respectively. The important finding from Table 1 is that the biases of IV2 and IV2-G1 are considerably smaller than the biases for the other estimators when $\delta_2 \neq 0$. Surprisingly, the IV estimator using the rank of B_i as an instrument for B_i does not perform any better than the OLS estimator. For the OLS and IV1 related estimators, (a) for $\delta_2 > 0$ as δ_2 increases, the biases increase; (b) for δ_2=0.5 and 1.0 as T increases, the biases fall; and (c) the impact of increasing N is mixed. Although not reported in detail to save space, a similar pattern of biases is observed for the corresponding estimators of δ_1.

Table 1: Bias of Estimators of δ_2

T=30		OLS	GLS1	GLS3	IV1	IV1-G1	IV1-G3	IV2	IV2-G1	IV2-G3
N	δ_2									
30	0.0	-0.0005	-0.0045	-0.0049	-0.0069	-0.0112	-0.0110	-0.0033	0.0010	-0.0112
	0.1	-0.0205	-0.0238	-0.0212	-0.0189	-0.0220	-0.0214	0.0001	-0.0040	-0.0203
	0.5	-0.0912	-0.0903	-0.0861	-0.0896	-0.0856	-0.0793	0.0216	0.0109	-0.0822
	1.0	-0.2291	-0.2136	-0.2028	-0.2239	-0.1966	-0.1887	0.0205	0.0198	-0.1872
60	0.0	0.0598	0.0601	0.0558	0.0618	0.0631	0.0617	-0.0076	-0.0069	0.0605
	0.1	-0.0536	-0.0456	-0.0445	-0.0412	-0.0383	-0.0378	0.0068	0.0063	-0.0378
	0.5	-0.1034	-0.1117	-0.1082	-0.0901	-0.0941	-0.0931	0.0047	0.0060	-0.0937
	1.0	-0.1798	-0.1833	-0.1746	-0.1644	-0.1690	-0.1639	0.0086	0.0053	-0.1669
100	0.0	0.0534	0.0488	0.0468	0.0432	0.0376	0.0368	0.0008	0.0012	0.0362
	0.1	-0.0398	-0.0331	-0.0332	-0.0440	-0.0406	-0.0407	0.0004	-0.0012	-0.0403
	0.5	-0.0969	-0.1017	-0.0972	-0.0819	-0.0859	-0.0851	0.0037	0.0044	-0.0849
	1.0	-0.2287	-0.2211	-0.2120	-0.2171	0.2131	-0.2061	-0.0021	-0.0002	-0.2079
T=60										
N	δ_2									
60	0.0	-0.0099	-0.0105	-0.0103	-0.0129	-0.0151	-0.0148	-0.0087	-0.0100	-0.0144
	0.1	-0.0333	-0.0272	-0.0266	-0.0302	-0.0260	-0.0255	0.0014	0.0007	-0.0257
	0.5	-0.0641	-0.0630	-0.0610	-0.0599	-0.0618	-0.0603	-0.0010	-0.0005	-0.0603
	1.0	-0.1372	-0.1328	-0.1298	-0.1266	-0.1240	-0.1220	-0.0002	-0.0017	-0.1230

Estimates of the type 1 errors for t-tests of the null hypothesis of $\delta_2 = 0$ for the OLS and IV2 estimators using various estimates of the covariance matrix are presented in Table 2. For IV2, in all but one case the estimated type I errors are not significantly different from the nominal size of the test. Despite the presence of heteroscedastic errors, t-tests based on an estimate of the covariance matrix assuming homoscedasticity (HOM) perform well. For the OLS estimator, test statistics computed using information about the known form of the heteroscedasticity (HET2 and HET3) always have type 1 errors close to their nominal size.

Rejection frequencies of the false null hypothesis of $\delta_2 = 0$ are presented in Table 3. For most combinations of N, T and δ_2, there is little difference in the rejection frequencies across the two estimators and the five estimates of the covariance matrix. This is perhaps a little surprising given the large differences in the biases of the OLS and IV2 estimators observed in Table 1. As is expected when δ_2 increases, the rejection frequencies increase. For δ_2=0.5 and 1.0 as T increases, the rejection frequencies fall. Again increases in N have a mixed impact.

Table 2: Type 1 Errors for t-tests of the Null Hypothesis $\delta_2 = 0$ (Nominal size =5%)

		OLS					IV2				
T	N	HOM	White	HET1	HET2	HET3	HOM	White	HET1	HET2	HET3
30	30	0.0698*	0.0758*	0.0626	0.0534	0.0508	0.0582	0.0726*	0.0606	0.0508	0.0460
30	60	0.0576	0.0662*	0.0642*	0.0574	0.0554	0.0448	0.0600	0.0634	0.0526	0.0468
30	100	0.0578	0.0620	0.0634	0.0556	0.0552	0.0438	0.0524	0.0576	0.0472	0.0466
60	60	0.0522	0.0610	0.0482	0.0454	0.0448	0.0518	0.0596	0.0508	0.0476	0.0466

Note: A * indicates the value is significantly different from 0.05.

Table 3: Rejection Frequencies for t-tests of the Null Hypothesis $\delta_2 = 0$ (Nominal size =5%)

		OLS					IV2				
T=30		HOM	White	HET1	HET2	HET3	HOM	White	HET1	HET2	HET3
N	δ_2										
30	0.1	0.0654	0.0772	0.0602	0.0536	0.0498	0.0540	0.0652	0.0596	0.0520	0.0476
	0.5	0.1328	0.1446	0.1274	0.1118	0.1048	0.1080	0.1292	0.1190	0.1056	0.0916
	1.0	0.3768	0.4078	0.4066	0.3890	0.3746	0.3822	0.4110	0.4322	0.4120	0.3944
60	0.1	0.0578	0.0682	0.0616	0.0546	0.0532	0.0488	0.0614	0.0578	0.0516	0.0466
	0.5	0.2228	0.2328	0.2398	0.2196	0.2128	0.2092	0.2140	0.2270	0.2096	0.1986
	1.0	0.6014	0.6032	0.6268	0.6046	0.6018	0.5224	0.5276	0.5418	0.5196	0.5136
100	0.1	0.0660	0.0668	0.0678	0.0598	0.0586	0.0620	0.0678	0.0702	0.0600	0.0578
	0.5	0.2984	0.2948	0.3148	0.2852	0.2828	0.2630	0.2646	0.2850	0.2570	0.2512
	1.0	0.8196	0.8168	0.8402	0.8234	0.8208	0.8160	0.8190	0.8350	0.8190	0.8164
T=60											
N	δ_2										
60	0.1	0.0558	0.0592	0.0564	0.0518	0.0512	0.0606	0.0690	0.0654	0.0590	0.0574
	0.5	0.2146	0.2186	0.2114	0.2024	0.2000	0.2048	0.2074	0.2040	0.1948	0.1900
	1.0	0.6930	0.6960	0.7100	0.6968	0.6908	0.7198	0.7154	0.7326	0.7204	0.7156

Rejection frequencies for t-tests of the true null hypothesis $\delta_1 = 0$ when the value of δ_2 is varied are displayed in Table 4. Since the null hypothesis is true, these rejection frequencies should be close to the nominal size of the test, 0.05. For many of the test statistics using the OLS estimator, it is found that the rejection frequencies are significantly higher than 0.05. For the IV estimates, the only estimates of the covariance matrix that consistently give test statistics with rejection frequencies that are not significantly different from 0.05 are HOM and HET3.

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Table 4: Rejection Frequencies for t-tests of the True Null Hypothesis $\delta_1 = 0$ (Nominal Size =5%)

		OLS					IV2				
T=30		HOM	White	HET1	HET2	HET3	HOM	White	HET1	HET2	HET3
N	δ_2										
30	0.0	0.0674*	0.0750*	0.0622	0.0546	0.0530	0.0638	0.0736*	0.0640	0.0544	0.0500
	1.0	0.0730*	0.0962*	0.0816*	0.0766*	0.0746*	0.0490	0.0684*	0.0664*	0.0578	0.0492
	5.0	0.2824*	0.3094*	0.4120*	0.3916*	0.2948*	0.0594	0.0724*	0.1460*	0.1344*	0.0504
60	0.0	0.0618	0.0686*	0.0614	0.0518	0.0502	0.0458	0.0612	0.0562	0.0476	0.0046
	1.0	0.0728*	0.0842*	0.0816*	0.0732*	0.0710*	0.0516	0.0606	0.0642*	0.0562	0.0490
	5.0	0.6646*	0.6998*	0.8350*	0.8232*	0.6898*	0.0530	0.0582	0.1794*	0.1676*	0.0466
100	0.0	0.0562	0.0618	0.0644*	0.0574	0.0568	0.0502	0.0576	0.0600	0.0522	0.0506
	1.0	0.1208*	0.1258*	0.1346*	0.1226*	0.1210*	0.0542	0.0566	0.0660*	0.0586	0.0522
	5.0	0.8422*	0.8658*	0.9410*	0.9324*	0.8684*	0.0430	0.0560	0.1562*	0.1396*	0.0480
T=60											
N	δ_2										
60	0.0	0.0572	0.0634	0.0534	0.0512	0.0506	0.0556	0.0622	0.0532	0.0508	0.0484
	1.0	0.0710*	0.0778*	0.0724*	0.0680*	0.0654*	0.0582	0.0652*	0.0614	0.0570	0.0512
	5.0	0.2044*	0.2214*	0.3068*	0.2966*	0.2128*	0.0476	0.0548	0.1000*	0.0958*	0.0430

Note: A * indicates the value is significantly different from 0.05.

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